

2016

(November)

PHYSICS

(Elective)

THIRD PAPER

(**Electricity and Magnetism**)

Full Marks : 75

Pass Marks : 25

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **all** questions

1. Choose the correct answer out of the given four alternatives and rewrite it : 1×5=5

(a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, the position vector of a point (x, y, z) and $r = |\vec{r}|$, then which of the following is incorrect?

(i) $\vec{\nabla} r = \frac{\vec{r}}{r}$

(ii) $\vec{\nabla} \cdot \vec{r} = 1$

(iii) $\nabla^2 (\vec{r} \cdot \vec{r}) = 6$

(iv) $\vec{\nabla} \times \vec{r} = 0$

(b) Which of the following is not a source of magnetostatic fields?

- (i) Permanent magnet
- (ii) An accelerated charge
- (iii) An electric field linearly changing with time
- (iv) A charged disc rotating at uniform speed

(c) The concept of displacement current was a major contribution attributed to

- (i) Faraday
- (ii) Lenz
- (iii) Maxwell
- (iv) Lorentz

(d) Which is not an example of convection current?

- (i) A moving charged belt
- (ii) Electronic movement in a vacuum tube
- (iii) An electron beam in a television tube
- (iv) Electric current flowing in a copper wire

(e) What happens when a steady potential difference is applied across the ends of a conducting wire?

- (i) All electrons move with constant velocity
- (ii) All electrons move with constant acceleration
- (iii) The random electronic motion will, on the average, be equivalent to a constant velocity of each electron
- (iv) The random electronic motion will, on the average, be equivalent to a non-zero constant acceleration of each electron

2. Answer the following questions :

1×8=8

(a) Consider the vector field

$$\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$$

Is the field solenoidal?

(b) Define a vector field.

(c) What is dielectric? Give an example.

- (d) Show that curl of the electric field \vec{E} is zero, if the field is conservative.
- (e) What is meant by uniqueness theorem?
- (f) Why no work is done in moving a test charge from one point to another on the surface of a conductor?
- (g) What do you mean by the relation $\vec{\nabla} \times \vec{B} = 0$?
- (h) Why is the series L-C-R circuit called an acceptor circuit?

3. Answer the following :

2×3=6

- (a) What do you mean by dielectric polarization?
- (b) What is the value of $\vec{\nabla} \times \vec{B}$ for points inside and outside a current loop?
- (c) A series L-C-R circuit has $L = 500 \mu\text{H}$, $C = 200 \mu\text{F}$ and $R = 32 \Omega$. The supply voltage is 5 volt. Calculate the resonant frequency and current at resonance.

4. Answer the following :

4×6=24

- (a) If $\vec{E} = (x+y)\hat{i} + (y-2x)\hat{j} - 2z\hat{k}$; prove that $\text{curl } \vec{E} = -3\hat{k}$ and $\text{div } \vec{E} = 0$.
- (b) Prove that for a linear, homogeneous and isotropic dielectric $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$. The symbols have their usual meaning.
- (c) Calculate the torque acting on a current loop placed in a uniform magnetic field.

Or

Explain the magnetic scalar and vector potentials.

- (d) A solenoid of 1200 turns is wound uniformly in single layer on a glass tube 24 cm long and 10 cm in diameter. Find the strength of the field—(i) at the centre and (ii) at one of the ends, when a current of 0.1 A flows through it.
- (e) A coil of inductance 0.5 H and resistance 500 ohm is connected in series with a capacitance of $1 \mu\text{F}$ and an alternating e.m.f. $10\sin 2000\pi t$. Find the current and phase difference between the e.m.f. and current.

- (f) Prove that Faraday's law of electromagnetic induction can be expressed in the differential form as

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The symbols have their usual meaning.

Or

Show that the magnetic energy U stored in a coil of self-inductance L is given by $\frac{1}{2}LI^2$, I being the current flowing in the coil.

5. State and prove Gauss' divergence theorem. 8

6. Write down Poisson's and Laplace's equations. Solve Laplace's equation using spherical polar coordinates. 2+6=8

Or

An uncharged conducting sphere of radius r is placed in an initially uniform electric field E_0 . Determine the potential and field at a point outside the sphere. 8

7. State and prove Ampere's circuital law of magnetic field. Express it in differential form. 1+5+2=8

(7)

8. Define Poynting vector for electromagnetic waves. State and derive Poynting theorem.

1+1+6=8

Or

Write Maxwell's equations in electromagnetic theory. Explain the physical significance of each of these equations.

2+6=8

2017

(November)

PHYSICS

(Elective)

THIRD PAPER

(Electricity and Magnetism)

Full Marks : 75

Pass Marks : 25

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer out of the given four alternatives and rewrite it : 1×5=5

(a) Which of the following is incorrect?

(i) $\vec{\nabla}(\vec{\nabla}\phi) = \nabla^2\phi$

(ii) $\vec{\nabla} \cdot (\vec{\nabla}\phi) = \nabla^2\phi$

(iii) $\vec{\nabla} \times \vec{\nabla}\phi = 0$

(iv) $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$

(b) When an electric dipole is placed in a uniform electric field, it experiences

- (i) both a net force and a torque
- (ii) only a net force
- (iii) only a torque
- (iv) neither a net force nor a torque

(c) Which of the following statements is not characteristic of a static magnetic field?

- (i) It is solenoidal
- (ii) It has no source or sink
- (iii) It is conservative
- (iv) Magnetic field lines are always closed

(d) Which one of the following expressions is not Maxwell's equation?

(i) $\vec{\nabla} \cdot \vec{D} = \rho_v$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(iii) $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$

(iv) $\vec{\nabla} \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(e) Which of the following is not true of a phasor?

- (i) It is a time dependent quantity
- (ii) It may be a scalar or a vector
- (iii) A phasor V_s may be represented as $V_0 e^{j\theta}$, where $V_0 = |V_s|$
- (iv) It is a complex quantity

2. Answer the following questions : 1×8=8

- (a) The electric field \vec{E} is zero at a point. Is the electric potential V necessarily zero at that point?
- (b) State Gauss theorem in electrostatics.
- (c) What will be the effect of introducing a dielectric slab between the plates of a capacitor?
- (d) What is a magnetic shell?
- (e) What do you mean by the relation $\vec{\nabla} \cdot \vec{B} = 0$?
- (f) Write down the magnetic moment of a circular current loop.
- (g) What is AC skin effect?
- (h) What do you understand by the time constant of an inductive circuit?

3. Answer the following questions :

2×3=6

- (a) For a position vector, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that

$$\vec{\nabla} \cdot (r^n \vec{r}) = (3+n)r^n$$

- (b) Write down the boundary conditions on \vec{E} and \vec{D} prevailing at the interface of two dielectrics, assuming no free charge to be present on the interface.
- (c) Find an expression for the potential energy stored in a charged capacitor.

4. Answer the following questions :

4×6=24

- (a) Find the Laplacian of the scalar field $W = 10r \sin^2 \theta \cos \phi$.

Or

A rigid body is rotating with constant angular velocity ω about a fixed axis. If v is the velocity of a point of the body, prove that $\vec{\nabla} \times \vec{v} = 2\omega$.

- (b) Prove that when a dielectric is polarized in an electric field, the volume charge density ρ_v is equal to the negative divergence of the polarization vector \vec{P} .

- (c) Show that the divergence of a magnetic field vector is zero everywhere.

Or

Explain why the relation $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and $\vec{\nabla} \cdot \vec{B} = 0$ are not sufficient to determine \vec{B} at a point even if \vec{J} is known at that point.

- (d) Show that the line integral of the magnetic field over a closed path is independent of the shape of the path.
- (e) In a CR circuit, if $C = 2.4 \mu\text{F}$, $R = 0.02 \text{ M}\Omega$, in what time will the charge in the capacitor attain half its final value? [Given : $\log_{10} 2 = 0.3010$]
- (f) Prove the reciprocity theorem $M_{12} = M_{21}$ for two coils 1 and 2 if M is the mutual inductance between the coils.

Or

Show that the wave equation for \vec{B} in a homogeneous linear isotropic medium with zero charge density and conductivity σ is given by .

$$\nabla^2 \vec{B} - \mu\sigma \frac{\partial \vec{B}}{\partial t} - \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

5. Answer the following questions :

8×4=32

(a) State and prove Stokes theorem in space.

2+6=8

(b) Give an outline of the multipole expansion of an electric field due to a charge distribution system and indicate what are monopole, dipole and quadrupole of the system.

5+3=8

Or

An uncharged dielectric sphere of radius a and dielectric constant K_1 is surrounded by a medium of dielectric constant K_2 . It is placed in a uniform magnetic field \vec{E}_0 . Find the electric potential inside the sphere. Hence, obtain the electric field inside the sphere.

6+2=8

(c) State Biot-Savart law. Find magnetic field at a distance z from a long straight wire carrying a steady current I . Apply this idea to find out the force of attraction between two long, parallel wires, a distance d apart, carrying currents I_1 and I_2 in the same direction.

1+4+3=8

Or

Discuss briefly the origin of hysteresis loop in a ferromagnetic material. Show that the energy dissipated per unit

2018

(November)

PHYSICS

(Elective)

THIRD PAPER

(Electricity and Magnetism)

Full Marks : 75

Pass Marks : 25

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer all questions

1. Choose the correct answer out of the given four alternatives and rewrite it : $1 \times 5 = 5$

(a) The curl of the gradient of a scalar field is

- (i) constant
- (ii) a vector
- (iii) a scalar
- (iv) zero

(b) The major contribution of magnetism in a substance is due to

- (i) orbital motion of electrons
- (ii) spin motion of electrons
- (iii) equal contribution from orbital and spin motion of electrons
- (iv) hidden magnets

(c) The force between two long and parallel wires carrying currents I_1 and I_2 , and separated by a distance R is proportional to

(i) $\frac{I_1 I_2}{R}$

(ii) $\frac{I_1 + I_2}{R}$

(iii) $\left(\frac{I_1 I_2}{R} \right)^2$

(iv) $\frac{I_1 I_2}{R^2}$

(d) Self-inductance per unit length of a long solenoid of radius R with n turns per unit length is

(i) $\mu_0 \pi R^2 n^2$

(ii) $2\mu_0 \pi R^2 n$

(iii) $2\mu_0 \pi R^2 n^2$

(iv) $\mu_0 \pi R^2 n$

(e) What is the approximate peak value of an alternating current producing four times the heat produced per second by a steady current of 2 A in a resistor?

(i) 2.8 A

(ii) 4 A

(iii) 5.6 A

(iv) 8 A

2. Answer the following questions :

1×8=8

(a) Define a vector field.

(b) What is a lamellar vector field?

(c) What do you mean by dielectric polarization?

- (d) What is an electric quadrupole?
- (e) Define magnetic susceptibility.
- (f) Explain why an isolated monopole does not exist.
- (g) Why is inductance called electrical inertia?
- (h) Why is the quantity $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ referred to as a displacement current?

3. Answer the following questions : 2×3=6

(a) What is an equipotential surface? Can two equipotential surfaces intersect?

(b) Consider the vector field $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. Is the field solenoidal?

(c) If $\vec{A} = 2x^2z^2\hat{i} - 2xy^2\hat{j} + 2x^2y^2\hat{k}$, then find the value of $\text{curl } \vec{A}$ at the point (1, 1, 1).

4. Answer the following questions : $4 \times 6 = 24$

(a) Three identical point charges $+Q$ each are located at the vertices of an equilateral triangle of side L . How much charge with type must be kept at the centroid of the system to vanish the potential of the new system?

(b) Apply Gauss' theorem to calculate the electric field due to an infinitely long uniformly charged straight wire.

(c) Differentiate between the terms scalar and vector potentials as applied in magnetism. Derive an expression for the vector potential and show that $\vec{B} = \vec{\nabla} \times \vec{A}$, where \vec{B} is the magnetic induction and \vec{A} is the magnetic vector potential.

(d) Show that the polarization volume charge density ρ_P is related to the polarization \vec{P} by the relation $\rho_P = -\vec{\nabla} \cdot \vec{P}$.

(e) From Faraday's laws of electromagnetic induction, deduce the relation
$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}.$$

- (f) A coil of self-inductance 0.7 henry is joined in series with a non-inductive resistance of 50 ohms. Calculate the wattless and power components as well as total current when connected to a supply of 200 V at 50 Hz. Also find the true power.

Or

A capacitor charged by a DC source through a resistance of $2\text{ M}\Omega$ takes 0.5 second to charge to $\frac{3}{4}$ of its final value. Find the capacitance of the capacitor.

5. Derive an expression for divergence of a vector field in Cartesian co-ordinates from first principles. What do you mean by solenoidal vector fields? 7+1=8

Or

State and prove Green's theorem in a plane. Express it in vector form. 5+3=8

6. Obtain the expression for local electric field at a point inside a dielectric. Hence, obtain Clausius-Mossotti relation. 5+3=8

(7)

Or

Calculate the force and torque on an electric dipole in an external electric field. Derive an expression for the potential energy when the dipole is placed in a uniform electric field. When is the energy of the dipole minimum?

$$2+2+3+1=8$$

7. State and prove Ampere's circuital law of magnetic field. Express it in differential form.

$$1+5+2=8$$

Or

Use Biot-Savart law to find an expression for magnetic field at a point on the axis of a current carrying solenoid. Hence, prove that the magnetic field at the end is half the magnetic field at the centre of a very long solenoid.

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8. Write down Maxwell's electromagnetic field equations explaining their physical significance.

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3rd Semester Exam., 2022 (November)

PHYSICS

(Elective)

THIRD PAPER

(Electricity and Magnetism)

Full Marks : 75

Pass Marks : 25

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer of the following :

1×10=10

(a) If $\vec{A} \cdot \vec{B} = 0$ and if \vec{A} and \vec{B} are not zero, then \vec{A} is

(i) parallel to \vec{B}

(ii) at an angle of 45° to \vec{B}

(iii) perpendicular to \vec{B}

(iv) at an angle of 60° to \vec{B}

(b) The curl of the gradient of a scalar field is

- (i) constant
- (ii) a vector quantity
- (iii) a scalar quantity
- (iv) zero

(c) The electrostatic potential at a distance r from an electric dipole varies as

- (i) r
- (ii) $\frac{1}{r}$
- (iii) $\frac{1}{r^2}$
- (iv) $\frac{1}{r^3}$

(d) The force on a closed current loop placed in a uniform magnetic field is

- (i) zero
- (ii) infinite
- (iii) finite
- (iv) None of the above

(e) The magnetic induction \vec{B} and the magnetic vector potential \vec{A} are related by

(i) $\vec{A} = \vec{\nabla} \times \vec{B}$

(ii) $\vec{\nabla} \times (\vec{A} \times \vec{B}) = 0$

(iii) $\vec{B} = \vec{\nabla} \times \vec{A}$

(iv) $\vec{B} = -\vec{\nabla} \times \vec{A}$

(f) The concept of displacement current was a major contribution attributed to

(i) Lenz

(ii) Faraday

(iii) Lorentz

(iv) Maxwell

(g) The SI unit of the electric polarization for a homogeneous isotropic dielectric is

(i) coulomb per square-metre

(ii) coulomb per cubic-metre

(iii) coulomb per square-centimetre

(iv) coulomb per cubic-centimetre

(h) The condition for oscillatory discharge in an L - C - R circuit is

(i) $R = 2\sqrt{\frac{L}{C}}$

(ii) $R = 0$

(iii) $R > 2\sqrt{\frac{L}{C}}$

(iv) $R < 2\sqrt{\frac{L}{C}}$

(i) Two parallel conductors carrying currents in opposite directions produce a force of

(i) repulsion

(ii) attraction

(iii) both repulsion as well as attraction

(iv) None of the above

(j) The Maxwell's equation which remains unchanged when a medium changes is

(i) $\vec{\nabla} \cdot \vec{B} = 0$

(ii) $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

(iii) $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

(iv) None of the above

2. Answer the following questions : 1×6=6

- (a) What is the directional derivative of a scalar field?
- (b) What is a lamellar vector field?
- (c) State uniqueness theorem.
- (d) What is zonal harmonics?
Or Define magnetic moment.
- (e) What is hysteresis loss?
- (f) Define Lorentz force.

3. Answer any *three* of the following questions : 2×3=6

- (a) Find the constant a for which the vector $\vec{A} = \hat{i}(x + 3y) + \hat{j}(y - 2z) + \hat{k}(x + az)$ is solenoidal.
- (b) Prove that the root-mean-square value of the alternating current is 0.71 times the peak value current.
- (c) Why are stranded insulator wires used in high frequency AC carrying devices?
- (d) Why is the parallel resonant circuit called a rejector circuit?
- (e) Show that the resonant frequency of a series resonant circuit depends on the product of L and C .

4. Answer any *five* of the following questions :

4×5=20

- (a) From Faraday's law of electromagnetic induction, deduce the relation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- (b) Prove that for a linear, homogeneous and isotropic dielectric, $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, where the symbols have their usual meanings.

- (c) Obtain the boundary conditions for \vec{B} and \vec{H} at the interface of two media of different magnetic permeabilities.

- (d) State Lenz's law and prove that Lenz's law is in accordance with the law of conservation of energy.

- (e) The electric potential in a region of space is given by

$$V = (5x - 7x^2y + 8y^2 + 16yz - 5z) \text{ volts}$$

where distances are measured in metre. Deduce an expression for the electric field intensity \vec{E} . Calculate Y-component of the field at the point (2, 4, -3).

- (f) A steady current of 20 ampere flows through a coil of inductance 0.03 henry when connected to a 200 volt DC supply. Calculate the current, power absorbed and power factor of the coil when connected to 200 volt, 50 cycles per second AC supply.
- (g) Show that the coefficient of self-induction L of a circuit is equal to twice the work required to establish unit current in the circuit.
5. (a) An alternating e.m.f. $= E_0 \sin \omega t$ is applied to the ends of a circuit containing a resistance R , a self-inductance L and a capacitance C . Calculate the current at any instant and deduce the condition under which electrical resonance occurs. 6+2=8
- Or
- (b) Derive the wave equations for \vec{E} and \vec{H} in a homogeneous linear isotropic medium with zero charge density and conductivity. Then, prove that the velocity of electromagnetic wave propagation V in free space is equal to the velocity of light c . 8
6. Obtain the multipole expansion of the potential due to a continuous volume charge distribution. Identify the monopole, dipole and quadrupole terms in the expression. 5+3=8

7. An uncharged conducting sphere of radius a is placed in an initially uniform electric field \vec{E}_0 . Determine the potential and field at a point outside the sphere. 6+2=8

8. (a) Derive the Clausius-Mosotti equation for an isotropic polarized dielectric. How can the atomic radius be determined from dielectric constant? 7+2=9

Or

- (b) Derive an expression for curl of a vector field in Cartesian coordinates from first principle. What do you mean by solenoidal vector fields? 7+2=9
