NAMBOL L. SANOI COLLEGE, NAMBOL

DEPARTMENT OF MATHEMATICS

QUESTION BANK FOR MATHEMATICS

PREVIOUS 4 YEARS (2016-2019)

SEMESTER IV

OBJECTIVE TYPE

(1 MARK QUESTIONS)

MATHEMATICS
(Elective)
FOURTH PAPER
(Mechanics)

2016

1. Choose and rewrite the correct answer for each of the following :

(1) The transverse component of acceleration of a moving particle is

(i) $\frac{1}{r}\frac{d}{dt}\left[r\frac{d\theta}{dt}\right]$ *(ii)* $\frac{1}{r}\frac{d}{dt}\left[r\frac{d^2\theta}{dt^2}\right]$

(iii) $\frac{1}{r}\frac{d}{dt}\left[r^2\frac{d\theta}{dt}\right]$ *(iv)* $\frac{1}{r^2}\frac{d}{dt}\left[r\frac{d\theta}{dt}\right]$

(2) A particle hanging from a fixed point by a light inextensible string of length l, is projected with an initial horizontal velocity u. The particle oscillates on either side of the lowest point if

(i) $u > \sqrt{5gl}$

(ii) $u \le \sqrt{2gl}$

(iii) $u \ge \sqrt{3gl}$

(iv) $\sqrt{2gl} < u < \sqrt{5gl}$

(3) With usual symbols, which one of the following is not true of a common catenary?

(i) $T \sin\psi = T_0$

(ii) $s = c\tan\psi$

(iii) $s = c\sinh\frac{x}{c}$

(iv) $y^2 = c^2 + s^2$

(4) The moment of inertia of a hollow sphere of radius a and mass M about the diameter is

(i) $\frac{Ma^2}{2}$ (ii) $\frac{Ma^2}{3}$

(iii) $\frac{3Ma^2}{2}$ (iv) $\frac{2Ma^2}{3}$

(5) The kinetic energy of a body moving in two dimensions (with usual symbols) is

(i) $\frac{1}{2}Mv^2$

(ii) $\frac{1}{2}Mk^2\left(\frac{d\theta}{dt}\right)^2$

(iii) $\frac{1}{2}Mv^2+\frac{1}{2}Mk^2\left(\frac{d\theta}{dt}\right)^2$

(iv) $\frac{1}{2}Mv^2+\frac{1}{2}M\left(\frac{d\theta}{dt}\right)^2$

(6) In the case of a plane lamina, if A and B are the moments of inertia about any two perpendicular lines lying in it, then the moment of inertia about a line through their intersection perpendicular to the plane is

(i) $A+B$

(ii) $A+AB+B$

(iii) A^2+B^2

(iv) A^2+AB+B^2

2017

(a) If the radial and transverse velocities of a particle are always proportional to each other, then its path is

(i) circle

(ii) parabola

(iii) catenary

(iv) equiangular spiral

(b) A particle slides down the outside of a smooth vertical circle of radius a, starting from rest at the highest point. The velocity with which the particle leaves the circle is

(i) $\frac{2ag}{3}$

(ii) $\frac{ag}{3}$

(iii) $\sqrt{\frac{2ag}{3}}$

(iv) $\frac{\sqrt{2ag}}{3}$

(c) The Cartesian equation of catenary of uniform strength is

(i) $y = a\cosh\left(\frac{x}{a}\right)$

(ii) $y = a\sec\psi$

(iii) $y = a\log\sec\left(\frac{x}{a}\right)$

(iv) $y = a\log(\tan\psi + \sec\psi)$

(d) The moment of inertia of a right circular cylinder about its axis is

(i) $\frac{1}{2}Ma^2$

(ii) $\frac{1}{3}Ma^2$

(iii) $\frac{1}{4}Ma^2$

(iv) $\frac{1}{5}Ma^2$

(e) The moment of momentum of a body rotating about a fixed axis is

(i) $Mk^2\left(\frac{d\theta}{dt}\right)^2$

(ii) $\frac{1}{2}Mk^2\left(\frac{d\theta}{dt}\right)^2$

(iii) $Mk^2\frac{d\theta}{dt}$

(iv) $\frac{1}{2}Mk^2\frac{d\theta}{dt}$

where the symbols have their usual meanings.

(f) The length of the simple equivalent pendulum of a circular disc of radius a about a horizontal axis a tangent to it, is

(i) $\frac{4}{5}a^2$

(ii) $\frac{5}{4}a^2$

(iii) $\frac{5}{6}a^2$

(iv) $\frac{6}{5}a^2$

2018

(a) The magnitude of the resultant acceleration of a particle moving in a plane curve at time t is

(i) $\left(\dfrac{d^2s}{dt^2}+\dfrac{v^2}{\rho}\right)^{1/2}$

(ii) $\left\{\left(\dfrac{ds}{dt}\right)^2+\left(\dfrac{v^2}{\rho}\right)^2\right\}^{1/2}$

(iii) $\left\{\left(\dfrac{d^2s}{dt^2}\right)^2+\left(\dfrac{v^2}{\rho}\right)^2\right\}^{1/2}$

(iv) $\left(\dfrac{d^2s}{dt^2}\right)^2+\left(\dfrac{v^2}{\rho}\right)^2$

(b) A particle slides down the arc of a smooth cycloid $S=4a\sin\psi$, whose axis is vertical and vertex downwards, then its period of oscillation about vertex is

(i) $\pi\sqrt{\dfrac{a}{g}}$

(ii) $2\pi\sqrt{\dfrac{a}{g}}$

(iii) $3\pi\sqrt{\dfrac{a}{g}}$

(iv) $4\pi\sqrt{\dfrac{a}{g}}$

(c) The moment of inertia of a solid sphere of mass M and radius a about a diameter is

(i) $\dfrac{1}{5}Ma^2$

(ii) $\dfrac{2}{5}Ma^2$

(iii) $\dfrac{3}{5}Ma^2$

(iv) $\dfrac{4}{5}Ma^2$

(d) A line OZ, taken as Z-axis, is a principal axis of a body and OX, OY are the other two axes. Then the inclination θ of one of the principal axes to OX is given by

(i) $\tan\theta = \dfrac{2F}{A-B}$

(ii) $\tan\theta = \dfrac{2F}{B-A}$

(iii) $\tan 2\theta = \dfrac{2F}{A-B}$

(iv) $\tan 2\theta = \dfrac{2F}{B-A}$

where the symbols have their usual meanings.

(e) If a uniform heavy string hangs freely under the action of gravity, then the equation of the curve is

(i) $y = c\sec\left(\dfrac{x}{c}\right)$

(ii) $y = c\cosh\left(\dfrac{x}{c}\right)$

(iii) $y = c\log\sec\left(\dfrac{x}{c}\right)$

(iv) $y = c\log\cos\left(\dfrac{x}{c}\right)$

(f) For a force system $(X, Y, Z; L, M, N)$, the pitch is

(i) $\dfrac{X^2+Y^2+Z^2}{LX+MY+NZ}$

(ii) $\dfrac{X^2+Y^2+Z^2}{LM+MN+NL}$

(iii) $\dfrac{LX+MY+NZ}{X^2+Y^2+Z^2}$

(iv) $\dfrac{LM+MN+NL}{X^2+Y^2+Z^2}$

2019

(a) A particle projected from the lowest point with a velocity u completes the circular motion along inside of a smooth circle of radius r, if

(i) $u^2 = 2gr$

(ii) $u^2 < 2gr$

(iii) $u^2 < 5gr$

(iv) $u^2 \geq 2gr$

(b) The radial component of acceleration is

(i) $\ddot{r} - \dot{r}\theta^2$

(ii) $\dot{r} - \ddot{r}\theta^2$

(iii) $\ddot{r} - \dot{r}\dot{\theta}^2$

(iv) $\ddot{r} - r\dot{\theta}^2$

where the symbols have their usual meanings.

(c) The number of quantities required to determine a wrench is

(i) 3

(ii) 4

(iii) 5

(iv) 6

(d) The moment of inertia of a circular ring of radius a and mass M about a line through its centre and perpendicular to its plane is

(i) Ma^2

(ii) $\frac{1}{2} Ma^2$

(iii) $\frac{1}{3} Ma^2$

(iv) $\frac{1}{4} Ma^2$

(e) The kinetic energy of a rigid body rotating about a fixed axis is

(i) $\frac{1}{2} MK \left(\frac{d\theta}{dt}\right)^2$

(ii) $\frac{1}{2} MK \frac{d^2\theta}{dt^2}$

(iii) $\frac{1}{2} MK^2 \left(\frac{d\theta}{dt}\right)^2$

(iv) $\frac{1}{2} MK^2 \frac{d^2\theta}{dt^2}$

(f) The minimum time of oscillation of a compound pendulum is

(i) $2\pi\sqrt{\frac{k}{g}}$

(ii) $2\pi\sqrt{\frac{2k}{g}}$

(iii) $2\pi\sqrt{\frac{3k}{g}}$

(iv) $2\pi\sqrt{\frac{k^2}{g}}$

VERY SHORT ANSWER TYPE

(1 MARK QUESTIONS)

2016

2. Write very short answer for each of the following questions :

(a) What is the terminal velocity of a particle of mass m falling under gravity in a resisting medium, the law of resistance being kmv^n ?

(b) Write the equation of motion when the mass of the moving particle varies.

(c) What is meant by a beat of a simple pendulum?

(d) Write the equation of a common catenary.

(e) Define a screw with respect to a system of forces acting on a rigid body.

(f) State the condition for a given system of forces to compound into a single force.

(g) What is a nul line?

(h) State Routh's rule for finding the moment of inertia of a body about an axis of symmetry.

(i) When are two mechanical systems said to be equimomental?

(j) State the principle of conservation of moment of momentum.

2017

(a) A point describes the cycloid $s = 4a\sin\psi$ with uniform speed v. Show that its acceleration at any point is $\frac{v^2}{\sqrt{(16a^2 - s^2)}}$.

(b) Give the geometrical representation of a simple harmonic motion.

(c) Write the equation of motion of a bead which moves on a smooth wire in a vertical plane under a resistance $\{= k(velocity)^2\}$.

(d) What is common catenary?

(e) Write the condition that the system of forces $(X, Y, Z; L, M, N)$ reduces to a single force.

(f) What is meant by a wrench?

(g) Define Poinsot's central axis.

(h) State d'Alembert's principle of rigid dynamics.

(i) What is momental ellipsoid?

(j) Define the centre of oscillation of a compound pendulum.

2018

(a) If the radial and transverse velocities of a particle are always proportional to each other, then show that the path is an equiangular spiral.

(b) Define the centre of suspension of a compound pendulum.

(c) What is the terminal velocity of a particle falling under gravity?

(d) What is an apse in a central orbit?

(e) Write the general equations of equilibrium of a string in a plane under the action of forces.

(f) What is the catenary of uniform strength?

(g) Define compound pendulum.

(h) What are the null lines of a given system of forces?

(i) State Routh's rule for finding the moment of inertia of a body about an axis of symmetry.

(j) State the condition for two bodies/systems to be equimomental.

2019

(a) Write the equation of motion of the body when mass varies.

(b) A particle falls under gravity in a resisting medium whose resistance is mkv^2, where m is its mass and v is the velocity. Find the terminal velocity.

(c) Define central orbit.

(d) Write the condition that the force system $(X, Y, Z; L, M, N)$ should reduce to a single force.

(e) Define Poinsot's central axis.

(f) What is common catenary?

(g) What is meant by pitch?

(h) State the theorem of parallel axes of moment of inertia.

(i) Define the centre of oscillation of a compound pendulum.

(j) State d'Alembert's principle of rigid body.

SHORT ANSWER TYPE

(3 MARKS QUESTIONS)

2016

3. Write short answer for each of the following questions :

(a) Show that the path of a point P which possesses two constant velocities u and v, the first of which is in a fixed direction and the second of which is perpendicular to the radius OP drawn from a fixed point O, is a conic whose focus is O and eccentricity is $\frac{u}{v}$.

(b) Find the intrinsic equation to a curve such that, when a point moves on it with constant tangential acceleration, the magnitudes of the tangential velocity and the normal acceleration are in a constant ratio.

(c) State Kepler's three laws of planetary motion.

(d) A particle moving with SHM in a straight line has velocities v_1, v_2 at distances x_1, x_2 from the centre of its path. Show that, if T be the period of its motion, then

$$T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(e) If three coplanar forces acting on a rigid body be in equilibrium, then prove that they must either all three meet at a point, or else all must be parallel to one another.

(f) Show that in the neighbourhood of the lowest point of a catenary the curve approximates in form to a parabola, while for large values of x, compared to c, the curve approximates in form to an exponential curve.

(g) Show that, whatever origin, or base point, and axes are chosen, for any given system of forces the quantities $X^2 + Y^2 + Z^2$ and $LX + MY + NZ$ are invariant.

(h) A uniform beam, of thickness $2b$, rests symmetrically on a perfectly rough horizontal cylinder of radius a. Show that the equilibrium of the beam will be stable or unstable according as b is less or greater than a.

(i) Find the moment of inertia of a circular disc of radius a and mass M about its diameter.

(j) Prove that the centres of suspension and oscillation of a compound pendulum are interchangeable.

(k) Show that the centre of inertia of a body moves as if all the mass of the body were collected at it, and as if all the external forces were acting at it in directions parallel to those in which they act.

(l) Derive the general form of equation of the momental ellipsoid of a body at a point O.

2017

(a) A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin. Prove that $\frac{dw}{dt} = -2w^2\cot\theta$, where $w = \frac{d\theta}{dt}$.

(b) A particle descends a smooth curve under the action of gravity, describing equal vertical distances in equal times, and starting in a vertical direction. Show that the curve is a semi-cubical parabola, the tangent at the cusp of which is vertical.

(c) Obtain the equation of a central orbit in polar form.

(d) State Kepler's three laws of planetary motion.

(e) Derive the equation of central axis of any given system of forces acting on a rigid body.

(f) A uniform beam of thickness $2b$ rests symmetrically on a perfectly rough horizontal cylinder of radius a. Show that the equilibrium of the beam will be stable or unstable according as b is less or greater than a.

(g) A rope of length $2l$ metres is suspended between two points at the same level, and the lowest point of the rope is b metres below the point of suspension. Show that the horizontal component of the tension is $\frac{w}{2b}(l^2 - b^2)$, where w being the weight of the rope per metre of its length.

(h) Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b.

(i) Show that the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

(j) Find the kinetic energy of a body moving in two dimensions.

(k) A point executes simple harmonic motion such that in two of its positions, the velocities are u and v and corresponding accelerations are α and β. Show that the distance between the positions is $\frac{v^2 - u^2}{\alpha + \beta}$.

(l) Prove that if a rigid body be moving under the action of external forces the sum of whose moments about a given line is zero throughout the motion, the angular momentum (or moment of momentum) of the body about that line remains unaltered throughout the motion.

(a) A particle moves in a curve $y = a \log \sec\left(\frac{x}{a}\right)$ in such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature.

(b) A point moving in a straight line with simple harmonic motion has velocities u_1 and u_2 when its distances from the centre are d_1 and d_2. Show that the period of motion is

$$2\pi\left(\frac{d_1^2 - d_2^2}{u_2^2 - u_1^2}\right)^{1/2}$$

(c) Find the equation of motion where the mass moving varies.

(d) Find the law of force, if a particle describes equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force P to the pole.

(e) Prove that if three coplanar forces acting on a rigid body be in equilibrium, they must either meet at a point or be parallel to one another.

(f) A heavy uniform string rests symmetrically on a smooth catenary whose axis is vertical and vertex upwards. Find the tension and pressure at any point.

(g) Find the form of the curve in which a chain hangs when the line density is given by $\frac{T_0}{ag}\sec^2\left(\frac{s}{a}\right)$, T_0 being the tension at the lowest point and s being measured from this point.

(h) Show that the quantities $(LX + MY + NZ)$ and $(X^2 + Y^2 + Z^2)$ are invariant for any given system of forces.

(i) Find the moment of inertia of a rectangular parallelopiped about an edge.

(j) Prove that the centres of suspension and oscillation of a compound pendulum are interchangeable.

(k) State and prove the principle of conservation of linear momentum under finite forces.

(l) A rough uniform board of mass M and length $2l$ rests on a smooth horizontal plane and a boy of mass m walks on it from one end to the other. Find the distance through which the board moves in this time.

2019

(a) Find the intrinsic equation to a curve such that when a point moves on it with constant tangential acceleration, the magnitude of the tangential velocity and normal acceleration are in a constant ratio.

(b) Prove that

$$\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}$$

where the symbols have their usual meanings.

(c) A particle is projected with velocity u along a smooth horizontal plane in a medium whose resistance per unit mass is λ times the cube of the velocity. Find its velocity.

(d) State Kepler's three laws of planetary motion.

(e) A uniform heavy string hangs freely under the action of gravity. Find the equation of the curve.

(f) Find the condition that a given system of forces should compound to a single force.

(g) Find the equation of the null plane of a given point (f, g, h) referred to rectangular axes OX, OY and OZ.

(h) A uniform cubical box of edge a is placed on the top of a fixed sphere, the centre of the face of the cube being in contact with the highest point of the sphere. What is the least radius of the sphere for which the equilibrium will be stable?

(i) Find the moment of inertia of a triangle ABC about a perpendicular to its plane through A.

(j) Prove that the centre of inertia of a body moves as if all the mass of the body were collected at it and as if all the external forces were acting at it in directions parallel to those in which they act.

(k) Find the kinetic energy of a body moving in two dimensions.

(l) Prove that the momental ellipsoid at the centre of an elliptic plate is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + z^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) = \text{constant}$$

(6 MARKS QUESTIONS)

2016

4. Answer any *two* parts : 6 mark each

(a) A particle slides down the outside of the arc of a smooth vertical circle, starting from rest at the highest point. Investigate the motion of the particle.

(b) Show that the accelerations along the tangent and normal to the path of a particle are $\frac{d^2s}{dt^2}\left(=v\frac{dv}{ds}\right)$ and $\frac{v^2}{\rho}$, where ρ is the radius of curvature of the curve at the point considered.

(c) A particle moves in a path so that its acceleration is always directed to a fixed point and is equal to $\frac{\mu}{(\text{distance})^2}$. Show that its path is a conic section and distinguish between the three cases that arise.

(d) Prove that the Cartesian equation of a catenary of uniform strength is of the form $y = a \log \sec \frac{x}{a}$.

(e) A lamina in the form of an isosceles triangle, whose vertical angle is α, is placed on a sphere, of radius r, so that its plane is vertical and one of its equal sides is in contact with the sphere. Show that, if the triangle be slightly displaced in its own plane, the equilibrium is stable if $\sin\alpha$ be less than $\frac{3r}{a}$, where a is one of the equal sides of the triangle.

(f) Three forces, each equal to P, act on a body; one at the point $(a, 0, 0)$ parallel to Oy, the second at the point $(0, b, 0)$ parallel to Oz, and the third at the point $(0, 0, c)$ parallel to Ox; the axes being rectangular, find the resultant wrench in magnitude and position.

(g) If the moments and products of inertia about any line, or lines, through the centre of inertia G of a body are known, obtain the corresponding quantities for any parallel line or lines.

(h) State D'Alembert's principle and derive the general equations of motion of a rigid body using the principle.

(i) If a rigid body swing, under gravity, from a fixed horizontal axis, show that the time of a complete small oscillation is

$$2\pi\sqrt{\frac{k^2}{hg}}$$

where k is its radius of gyration about the fixed axis, and h is the distance between the fixed axis and the centre of inertia of the body.

(j) A seconds pendulum gains 18 seconds a day at sea level. To what height must it be elevated in order to keep true time? (Assume the radius of the earth to be 6400 km.)

Or

A particle is projected under gravity and a resistance equal to mk (velocity) with a velocity u at an angle α to the horizon. Find the equation to the path of the particle and also find the greatest height attained.

(k) Prove that the necessary and sufficient conditions that a system of coplanar forces acting on a rigid body may be in equilibrium are that *(i)* the algebraic sum of the resolved parts of the forces in any two mutually perpendicular directions should be separately zero, and *(ii)* the algebraic sum of the moments of the forces about any point in their plane should also be zero.

Or

Show that any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis is along the direction of the force.

(a) Show that the acceleration $\vec{a}$ of a particle which moves along a plane curve with speed v is given by $\vec{a} = \dot{v}\hat{t} + \frac{v^2}{\rho}\hat{n}$, where $\hat{t}$ is the unit tangent vector, $\hat{n}$ is the unit vector along the normal and ρ, the radius of curvature at the point considered.

(b) A particle is tied to a string of length a and is projected from its lowest point, so that after leaving the circular path it describes a free path passing through the lowest point. Prove that the velocity of projection is $\sqrt{\left(\frac{7}{2}ag\right)}$.

(c) A heavy particle is projected vertically upwards with velocity u in a medium, the resistance of which is $gu^{-2}\tan^2\alpha$ times the square of the velocity, α being a constant. Show that the particle will return to the point of projection with velocity $u\cos\alpha$ after a time $ug^{-1}\cot\alpha\left(\alpha + \log\frac{\cos\alpha}{1-\sin\alpha}\right)$.

(d) State and prove the necessary and sufficient conditions of equilibrium of a system of coplanar forces acting on a rigid body.

(e) Prove that the Cartesian equation of common catenary is of the form $y = a\cosh(\frac{x}{a})$.

(f) A uniform beam of length $2a$ rests with its ends on two smooth planes which intersect in a horizontal line. If the inclination of the planes to the horizontal are α and β $(\alpha > \beta)$, show that the inclination θ of the beam to the horizontal in one of the equilibrium positions is given by $\tan\theta = \frac{1}{2}(\cot\beta - \cot\alpha)$.

(g) State and prove the theorem of parallel axes on a rigid body.

(h) Prove that the momental ellipsoid at a point on the circular edge of a solid cone is

$$(3a^2 + 2h^2)x^2 + (23a^2 + 2h^2)y^2 + 26a^2z^2 - 10ahxy = \text{constant}$$

where h is the height and a, the radius of the base.

(i) AB, BC are two equal similar rods freely hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B.

(j) A particle is projected vertically upwards under gravity in a resisting medium under resistance varies as the square of the velocity. Prove that the greatest height it has described in time t is $\frac{V^2}{g}\log\sec\left(\frac{gt}{V}\right)$, where V is the terminal velocity of the particle.

Or

A spherical raindrop, falling freely, receives in each instant an increase of volume equal to λ times its surface at that instant. Find the velocity at the end of time t and the distance fallen through in that time.

(k) Derive the condition of equilibrium of a string in a plane under the action of given forces.

2018

(a) Obtain the radial and transverse components of velocity and acceleration of a particle moving in a plane curve.

Or

Obtain the expressions for velocity, acceleration and position of a particle executing simple harmonic motion at any instant time.

(b) The base of a rough cycloidal arc is horizontal and its vertex downwards, a bead slides along it starting from rest at the cusp and coming to rest at the vertex. Show that $\mu^2 e^{\mu\pi} = 1$.

Or

A particle is projected with the velocity V from the cusp of a smooth inverted cycloid down the arc. Show that the time of reaching the vertex is

$$2\sqrt{\frac{a}{g}}\tan^{-1}\left[\frac{\sqrt{4ag}}{V}\right]$$

(c) The resistance of the air to a particle's motion is n times its weight and the particle is projected horizontally with velocity V. Show that the velocity of the particle, when it is moving at an inclination ϕ to the horizontal, is

$$V(1-\sin\phi)^{\frac{n-1}{2}}(1+\sin\phi)^{\frac{n+1}{2}}$$

Or

A particle describes an ellipse about a centre of force at the focus. Show that at any point of its path, the angular velocity about the other focus varies inversely as the square of the normal at that point.

(d) Forces equal to $3P$, $7P$ and $5P$ act along the sides AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and line of action of the resultant.

Or

A heavy uniform chain rests on a rough cycloid whose axis is vertical and vertex upwards, one end of the chain being at the vertex and the other at the cusp, if the equilibrium be limiting, show that

$$(1+\mu^2)e^{\frac{\mu\pi}{2}} = 3$$

(e) Prove that any system of forces acting on a rigid body can be reduced to a single force together with a couple whose axis is along the direction of the force.

Or

Equal forces act along the axes and along the straight line

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

Find the equations of the central axis of the system.

(f) A solid homogenous hemisphere of radius a has a solid right cone of the same substance constructed on its base, the hemisphere rests on the convex side of a fixed sphere of radius b, the axis of the cone being vertical. Show that the greatest height of the cone consistent with stability for a small rolling displacement is

$$\frac{a}{a+b}[\sqrt{(3b+a)(b-a)} - 2a]$$

Or

A uniform heavy bar AB can move freely in a vertical plane about a hinge at A and has a string attached to its end B which after passing over a small pulley at a point C vertically above A is attached to a weight. Show that the position of equilibrium in which AB is inclined to the veritical is an unstable one.

(g) Show that the momental ellipsoid at a point on the rim of a hemisphere is

$$2x^2 + 7(y^2 + z^2) - \frac{15}{4}xz = \text{constant}$$

(h) State and prove d'Alembert's principle.

(i) AB, BC are two equal similar rods freely hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B.

2019

4\. *(a)* Prove that for a particle moving along a plane curve at the point (r, θ), its acceleration $\vec{a}$ is

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\hat{e}_\theta$$

where $\dot{r} = \frac{dr}{dt}$ etc., and $\hat{e}_r$, $\hat{e}_\theta$ are unit vectors along and perpendicular to the radius vector of the particle.

Or

If in a simple harmonic motion (SHM), the velocities at distances a, b, c from a fixed point on the straight line which is not the centre of force be u, v, w respectively, then show that the periodic time T is given by the equation

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

(b) A particle descends a smooth curve under the action of gravity, describing equal vertical distances in equal times and starting in a vertical direction. Show that the curve is a semi-cubical parabola, the tangent at the cusp of which is vertical.

Or

A particle is projected along inside of a smooth vertical circle of radius a, from the lowest point. Show that the velocity of projection, so that after leaving the circle the particle may pass through the centre is

$$\left(\sqrt{\frac{ag}{2}}\right)(\sqrt{3}+1)$$

(c) A spherical raindrop of radius r falls from rest through a vertical height h receiving throughout the motion an accumulation of condensed vapour at the rate of K g-m per sq.cm per sec, no vertical force but gravity acting. Show that when it reaches the ground its radius will be

$$K\sqrt{\frac{2h}{g}}\left(1+\sqrt{1+\frac{gr^2}{2hK^2}}\right)$$

6

Or

A particle of mass m is falling under the influence of gravity through a medium whose resistance equals λ times the velocity. If the particle is released from rest, show that the distance fallen through in time t is

$$g\frac{m^2}{\lambda^2}\left(e^{-\frac{\lambda t}{m}}+\frac{\lambda t}{m}-1\right)$$

6

(d) State and prove the necessary and sufficient condition of equilibrium of a system of coplanar forces acting on a rigid body.

Or

Define catenary of uniform strength. Derive the equation of catenary of uniform strength.

(e) Forces X, Y, Z act along the three lines given by the equations $y=0$, $z=c$; $z=0$, $x=a$; $x=0$, $y=b$; prove that the pitch of the equivalent wrench is

$$\frac{(aYZ+bZX+cXY)}{X^2+Y^2+Z^2}$$

If the wrench reduces to a single force, prove that the line of action of the force lies on the hyperboloid

$$(x-a)(y-b)(z-c)=xyz$$

Or

Forces X, Y, Z act along the three straight lines $y = b$, $z = -c$; $z = c$, $x = -a$; $x = a$, $y = -b$. Show that they will have a single resultant if

$$\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$$

and that the equations of its line of action are any two of the three

$$\frac{y}{Y} - \frac{z}{Z} - \frac{a}{X} = 0, \quad \frac{z}{Z} - \frac{x}{X} - \frac{b}{Y} = 0,$$

$$\frac{x}{X} - \frac{y}{Y} - \frac{c}{Z} = 0$$

(f) A body consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table, the hemisphere being in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere.

Or

A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is unstable.

(g) Find the principal axes of a right circular cone at a point on the circumference of the base, and show that one of them will pass through its centre of gravity if the vertical angle of the cone is $2\tan^{-1}\left(\frac{1}{2}\right)$.

(h) Derive the general equations of motion of a rigid body from d'Alembert's principle.

(i) A solid homogeneous cone of height h and vertical angle 2θ oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is $\frac{h}{5}(4 + \tan^2\theta)$.