NAMBOL L. SANOI COLLEGE, NAMBOL

DEPARTMENT OF MATHEMATICS

QUESTION BANK FOR MATHEMATICS

PREVIOUS 4 YEARS (2016-2019)

SEMESTER II

OBJECTIVE TYPE

(1 MARK QUESTIONS)

2016

MATHEMATICS

(Elective)

SECOND PAPER

(Calculus and Ordinary Differential Equations)

1. Choose and write the correct answer for each of the following :

(a) If

$$\lim_{x \to 0} \frac{\sin 3x + a\sin 2x}{x^2}$$

be finite, then the value of a is

(i) $\frac{3}{2}$ (ii) $-\frac{3}{2}$

(iii) $\frac{2}{3}$ (iv) $-\frac{2}{3}$

(b) The curve for which the curvature is zero at every point is

(i) an ellipse

(ii) a parabola

(iii) a hyperbola

(iv) a straight line

(c) The perimeter of the curve $r = a\cos\theta$ is

(i) $\frac{1}{2}\pi a$

(ii) $\frac{1}{3}\pi a$

(iii) $\frac{1}{4}\pi a$

(iv) πa

(d) The volume V of the solid of revolution, where $x = f(y)$ is the curve, the axis of revolution is the y-axis, bounded by $y = y_1$ and $y = y_2$, is given by

(i) $V = \pi \int_{y_1}^{y_2} x\, dy$

(ii) $V = 2\pi \int_{y_1}^{y_2} x^2 dy$

(iii) $V = \pi \int_{y_1}^{y_2} x^2 dy$

(iv) $V = 2\pi \int_{y_1}^{y_2} x\, dy$

(e) The integrating factor of the equation

$$\frac{dy}{dx} + \frac{2y}{x} = e^x$$

is

(i) x^2 (ii) e^{2x}

(iii) $\log 2x$ (iv) $\frac{2}{x}$

(f) The solutions of the simultaneous equations

$$\frac{3dy}{dx} = \frac{y}{x}; \quad \frac{2dz}{dx} = \frac{z}{x}$$

are

(i) $xy = c_1$ and $yz = c_2$

(ii) $xy^3 = c_1$ and $y^3 z^2 = c_2$

(iii) $x = c_1 y^3$ and $y^3 = c_2 z^2$

(iv) $x^3 = c_1 y$ and $y^2 = c_2 z^3$

(g) The particular integral of

$$\sec x \frac{d^2y}{dx^2} - 4(\sec x) y = 2\sin x$$

is

(i) $\frac{1}{8}\sin 2x$ (ii) $-\frac{1}{8}\sin 2x$

(iii) $\frac{1}{8}\cos 2x$ (iv) $-\frac{1}{8}\cos 2x$

2017

(a) The reduction formula for $I_n = \int_0^{\pi/2} \sin^n x\, dx$ is given by

(i) $I_n = \frac{n}{n-1} I_{n-2}$

(ii) $I_n = \frac{n-1}{n} I_{n-2}$

(iii) $I_n = \frac{n-1}{n} I_{n-1}$

(iv) $I_n = \frac{n}{n-1} I_{n-1}$

(b) If $u = \sin\left(\frac{x^2 + y^2}{xy}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(i) 0 (ii) 1

(iii) −1 (iv) 2

(c) The area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is

(i) $\frac{3}{4}\pi a^2$ (ii) $\frac{4}{3}\pi a^2$

(iii) $\frac{3}{8}\pi a^2$ (iv) $\frac{8}{3}\pi a^2$

(d) The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ revolves about its minor axis. The volume of the solid of revolution is

(i) $\frac{4}{3}\pi ab^2$ (ii) $\frac{3}{4}\pi a^2 b$

(iii) $\frac{4}{3}\pi a^2 b$ (iv) $\frac{3}{4}\pi ab^2$

(e) The condition of exactness of the differential equation $Mdx + Ndy = 0$ is

(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$ (iv) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(f) The differential equation for which $y = (C_1 x^2 + C_2 x + C_3) e^{2x}$ is the general solution, is

(i) $(D-2)y = 0$ (ii) $(D-3)y = 0$

(iii) $(D-2)^3 y = 0$ (iv) $(D-3)^2 y = 0$

(g) If $f(x) = Ax^2 + Bx + C$, then the value of ξ in the mean value theorem $f(b) - f(a) = (b-a)\, f'(\xi)$ is

(i) $b - a$ (ii) $a + b$

(iii) $\frac{b-a}{2}$ (iv) $\frac{a+b}{2}$

2018 (May)

a) The value of $\int_0^{\infty} e^{-x} dx$ is

i) 0 ii) 1
iii) e iv) ∞

b) Perimeter of the asteroil $x^{2/3} + y^{2/3} = a^{2/3}$ is

i) 2a ii) 3a
iii) 4a iv) 6a

c) The circle $x^2 + y^2 = a^2$ revolves about the x – axis. The surface area of the whole sphere generated is

i) πa^2 ii) $2\pi a^2$
iii) $4\pi a^2$ iv) $6\pi a^2$

d) Integrating factor of the differential equation

$\cos^2 x \frac{dy}{dx} + y = \tan x$ is

i) $\tan x$ ii) $-\tan x$
iii) $e^{\tan x}$ iv) $e^{-\tan x}$

e) If $v = \log \frac{x^2 + y^2}{x + y}$, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}$ equals

i) 0 ii) 1
iii) 2 iv) 3

3.

f) Particular integral of $(D-2)^2 y = x^3 e^{2x}$ is

i) $\frac{1}{20}x^5$ ii) $\frac{1}{20}x^5 e^{2x}$
iii) $\frac{1}{4}x^4 e^{2x}$ iv) $(Ax + B)e^{2x}$

g) If $\lim_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ be finite, then the value of a is

i) 0 ii) 1
iii) 2 iv) −1

2019

(a) The area of the hypocycloid $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ is

(i) $\frac{1}{4}\pi ab$ (ii) $\frac{1}{8}\pi ab$

(iii) $\frac{3}{8}\pi ab$ (iv) $\frac{8}{3}\pi ab$

(b) The circle $x^2 + y^2 = a^2$ revolves about the y-axis. The volume of the solid of revolution is

(i) πa^3

(ii) $\frac{1}{3}\pi a^3$

(iii) $\frac{2}{3}\pi a^3$

(iv) $\frac{4}{3}\pi a^3$

(c) If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(i) 0

(ii) 1

(iii) 2

(iv) −1

(d) The reduction formula for $I_n = \int_0^{\pi/4} \tan^n x\,dx$ is given by

(i) $I_n = \frac{1}{n-1} - I_{n-2}$

(ii) $I_n = \frac{1}{n-2} - I_{n-1}$

(iii) $I_n = \frac{1}{n+1} - I_{n-1}$

(iv) $I_n = \frac{1}{n+2} - I_{n-2}$

(Continued)

(e) Particular integral of the differential equation $(D-1)^2 y = e^x$ is

(i) e^x

(ii) xe^x

(iii) $x^2 e^x$

(iv) $\frac{1}{2}x^2 e^x$

(f) The condition for exactness of the differential equation $Mdx + Ndy = 0$ is

(i) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(ii) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(iii) $\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$

(iv) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$

(g) The value of $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is

(i) 0

(ii) π

(iii) $\pi/2$

(iv) ∞

VERY SHORT ANSWER TYPE

(1 MARK QUESTIONS)

2016

2. Write very short answer for each of the following questions :

(a) State Cauchy's mean value theorem.

(b) In the mean value theorem

$$f(b) = f(a) + (b-a) f'(c)$$

If $f(x) = 2x^2$, $a = 0$ and $b = 2$, then what is the value of c?

(c) Write Taylor's finite series expansion for e^{a+n}, with remainder after n terms.

(d) Give reason why the mean value theorem is not applicable to the function $f(x) = |x|$ in the interval $[-2, 2]$.

(e) State the reason for the failure of applicability of the Euler's theorem on the function $\sin(x+y)$.

(f) The interval $[a, b]$ is divided into n equal subintervals, each of length h by the points $x_0 = a$, $x_1, x_2, \cdots, x_n = b$. Represent $\int_a^b e^{kx} dx$ as limit of sum, k being a constant.

(g) Write down the reduction formula for

$$I_n = \int_0^{\pi/2} \cos^n x \, dx$$

(h) Show in a diagram the region of integration of the integral

$$\int_0^2 dx \int_x^{\sqrt{2x}} f(x, y) \, dy$$

(i) Evaluate :

$$\int_0^1 \int_0^y 8xy \, dx \, dy$$

(j) Is the differential equation

$$(e^x + xy^2) dx = x^2 y dx$$

exact?

(k) What is the complementary function of $(D^4 + 2D^2 + 1) y = \sin 2x$?

(l) What is the particular integral of $(D^2 + 4) y = e^{-2x}$?

2017

(a) Is Lagrange's mean value theorem applicable to the function $f(x) = |x|$ in the interval $[-1, 1]$? Give reason of your answer.

(b) Write Cauchy's remainder after n terms in the expansion of $\sin x$.

(c) Evaluate $\frac{1}{(D-3)^2} e^{3x}$.

(d) State L' Hospital's theorem.

(e) Write Maclaurin's series in finite form.

(f) Find the radius of curvature of the curve

$$s = a \log \tan\left(\frac{\pi}{4} + \frac{\psi}{2}\right)$$

at the point (s, ψ).

(g) Express $\int_a^b f(x)\, dx$ as the limit of a sum.

(h) If $x = r\cos\theta$ and $y = r\sin\theta$, then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(i) Show in a diagram the region of integration of the integral

$$\int_0^1 dx \int_x^{1/x} \frac{y\,dy}{(1+xy)^2(1+y^2)}$$

(j) Write down the complete primitive of Clairaut's equation $y = px + f(p)$ where $p = \frac{dy}{dx}$.

(k) Define singular solution of a differential equation.

(l) What is meant by a singular point of the differential equation

$$f(x)\frac{d^2y}{dx^2} + g(x)\frac{dy}{dx} + r(x)\, y = 0 \; ?$$

2018

a) Is Rolle's Theorem applicable to the function $f(x) = \tan x$ in the interval $[0, \pi]$? Give reason of your answer.

b) State Cauchy's mean value theorem.

c) What is meant by an indeterminate form?

d) If $f(x) = e^x$, find the value of c in the mean value theorem $f(b) - f(a) = (b-a)f'(c)$, where $a = 0$ and $b = 1$.

e) Evaluate $\int e^{ax} \cos bx dx$.

f) Express $\int_0^1 f(x)dx$ as the limit of a sum.

g) If $x = u(1+v)$ and $y = v(1+u)$, find $\frac{\partial(x,y)}{\partial(u,v)}$.

h) Write down the expression for the radius of curvature of the Cartesian equation $y = f(x)$.

i) Change the order of integration in the integral $\int_0^1 dx \int_x^{\sqrt{x}} f(x,y)dy$.

j) What is the condition for integrability of the total differential equation $Pdx + Qdy + Rdz = 0$?

k) Give the geometrical interpretation of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

l) What is meant by a homogeneous linear equation?

2019

(a) Write down the general solution of the differential equation $(D+2)^3 y = 0$.

(b) Give geometrical interpretation of Lagrange's mean value theorem.

(c) Prove by the method of summation, that

$$\int_a^b dx = b - a$$

(d) Write down Maclaurin's series in finite form.

(e) If $f(x) = Ax^2 + Bx + C$, then find the value of ξ in the mean value theorem

$$f(b) - f(a) = (b-a)f'(\xi)$$

(f) Write down the complete primitive of Clairaut's equation $y = px + f(p)$, where $p = \frac{dy}{dx}$.

(g) Show in a diagram the region of integration of the integral

$$\int_0^2 dx \int_x^{\sqrt{x}} f(x, y)\,dy$$

(h) What is meant by the curvature of a curve?

(i) Evaluate $\int_0^1 \int_0^y xy\,dx\,dy$.

(j) If $x = r\cos\theta$ and $y = r\sin\theta$, then find

$$\frac{\partial(x, y)}{\partial(r, \theta)}$$

(k) Give the geometrical interpretation of

$$Pdx + Qdy + Rdz = 0$$

(l) State L' Hospital's theorem.

SHORT ANSWER TYPE

(3 MARKS QUESTIONS)

2016

3. Write short answer for each of the following questions :

(a) Using ε-δ definition of continuity, show that

$$f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$.

(b) If $x = \tan(\log y)$, then prove that

$$(1 + x^2)y_{n+2} + (2nx + 2x - 1)y_{n+1} + n(n+1)y_n = 0$$

(The symbols have their usual meanings.)

(c) Verify Rolle's theorem for the function $f(x) = e^{-x}\sin x$ in the interval $[0, \pi]$.

(d) By using Maclaurin's theorem, expand $(1 + x)^m$, where $x \in \mathbb{R}$ and m is a positive integer.

(e) By using L' Hospital's rule, show that

$$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{1/x} = 1$$

(f) If $H = f(y-z, z-x, x-y)$, then prove that

$$\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$$

(g) Find the radius of curvature at the point t on the parabola $x = 2at$, $y = at^2$.

(h) If f is integrable on $[a, b]$ $(a < b)$, and if there exists a function of such that $g'(x) = f(x) \quad \forall x \in [a, b]$, then prove that

$$\int_a^b f(x)\,dx = g(b) - g(a)$$

(i) Determine the length of an arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ measured from the vertex. Hence or otherwise obtain the length of a complete cycloid.

(j) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then show that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$$

(k) Evaluate the integral

$$I = \int_0^1 dx \int_0^x \sqrt{x^2 + y^2}\, dy$$

by passing or to the polar coordinates.

(l) Prove that the singular solution of

$$z = px + \frac{a}{p};\quad p = \frac{dz}{dx}$$

is the parabola $z^2 = 4ax$.

(m) Find the orthogonal trajectories of the family of straight lines $w = mv$, where m is the parameter.

(n) Solve :

$$x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 9y = 0$$

(o) Discuss the nature of the points $x = 1$ and $x = -1$ of

$$(x+1)(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

Also, if it is a singular point, state whether regular or irregular.

2017

3. Write short answer for each of the following questions : 3×15=45

(a) Using (ε-δ) definition of limit, prove that

$$\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$$

(b) If $y = \tan^{-1} x$, then prove that

$$(1+x^2)\, y_{n+1} + 2nxy_n + n(n-1)\, y_{n-1} = 0$$

(c) Prove that

$$\lim_{x\to 0} \frac{(1+x)^{1/x} - e}{x} = -\frac{1}{2}e$$

(d) Show that

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

does not exist.

(e) If $v = f(u)$, u being a homogeneous function of degree n in x and y, then show that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = nu\frac{dv}{du}$$

(f) Show that the function $f(x, y) = 5x^4 + 3x^2y + y^2$ has a minimum at (0, 0).

(g) Find the asymptotes of the curve $x^3 + 2x^2y + xy^2 - x + 1 = 0$.

(h) If $u_n = \int_0^{\pi/2} \theta \sin^n \theta \, d\theta$, $n > 1$, then prove that

$$u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$$

(i) Find the length of the arc of the curve

$$x = e^{\theta} \sin\theta$$
$$y = e^{\theta} \cos\theta$$

(j) Prove that

$$\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3}\, dy = \frac{1}{2}$$

(k) Show that the volume of a right circular cone of height h and base of radius r is $\frac{1}{3}\pi r^2 h$.

(l) **Solve :**

$$\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$$

(m) **Show that the solution of**

$$\frac{d^2y}{dx^2} + k\frac{dy}{dx} + \mu y = 0$$

is $y = e^{-\frac{kx}{2}}(A\cos nx + B\sin nx)$ if $k^2 < 4\mu$ and $n^2 = \mu - \frac{1}{4}k^2$.

(n) Find the singular solution of the equation

$$y = px + \sqrt{a^2p^2 + b^2}$$

where $p = \frac{dy}{dx}$.

(o) Find the particular integral of $(D^2 - D - 2)y = \sin 2x$.

2018

Answer each of the following questions: 3×5=15

a) Using $\in - \delta$ definition of continuity, prove that the function $f(x) = x^2 + 2x - 1$ is continuous at $x = 2$.

b) If $y = \sin(m\sin^{-1}x)$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

c) Evaluate: $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$.

d) If u be a homogeneous function of x and y of degree n, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$.

e) Find $f_{xy}(0,0)$ for the function f given by

$$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2},\ (x,y) \neq (0,0)$$

$$f(0,0) = 0.$$

f) Show that the asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ form a square of side $2a$.

g) Show that the function $f(x, y) = 3x^3 + 4x^2y - 3xy^2 - 4y$ has neither a maximum nor a minimum at the origin.

h) If $I_n = \int_0^{\pi/4} \tan^n\theta\, d\theta$, prove that $n(I_{n+1} + I_{n-1}) = 1$.

i) Obtain the intrinsic eauation of the catenary $y = c\cos\frac{x}{c}$ in the form $s = c\tan\psi$.

j) Evaluate the integral $\int_0^1 dx\int_0^x \sqrt{x^2+y^2}\,dy$ by passing on to the polar coordinates.

k) Prove that the volumes of the ellipsoid formed by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the major axis is $\frac{4}{3}\pi ab^2$.

l) Solve: $\frac{adx}{(b-c)yz} = \frac{bdy}{(c-a)zx} = \frac{cdz}{(a-b)xy}$.

m) Find the complete primitive of $p^2+2xp-3x^2=0$, where $p = \frac{dy}{dx}$.

n) Solve: $(D^2 - 2mD + m^2 + n^2)y = 0$.

o) Solve: $(D^2 - 4)y = \sin 2x$.

2019

(a) Using ε-δ definition of limit, prove that

$$\lim_{x \to 2} (x^2 - 3x + 5) = 3$$

(b) If $u = \phi(H_n)$, where H_n is a homogeneous function of degree n in x, y, z, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = n\frac{F(u)}{F'(u)}$, where $F(u) = H_n$.

(c) If $u_n = \int_0^{\pi/2} x^n \sin x \, dx$, $(n \geq 1)$, then show that $u_n + n(n-1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$.

(d) Find the singular solution of the equation $y = px + \frac{a}{p}$, where $p = \frac{dy}{dx}$.

(e) Show that the volume of a right circular cone of height h and base of radius r is

$$\frac{1}{3}\pi r^2 h$$

(f) Solve :

$$\frac{dx}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2 + y^2}$$

(g) Show that if $l\frac{d^2\theta}{dt^2} + g\theta = 0$, and if $\theta = \alpha$ and $\frac{d\theta}{dt} = 0$ when $t = 0$, then

$$\theta = \alpha \cos\{t\sqrt{g/l}\}$$

(h) Solve :

$$\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$$

(i) Find the perimeter of the astroid

$$x^{2/3} + y^{2/3} = a^{2/3}$$

(j) Evaluate :

$$\lim_{x \to 0} (\cos x)^{1/x}$$

(k) Show that $\lim_{(x,\, y) \to (0,\, 0)} xy\frac{x^2 - y^2}{x^2 + y^2} = 0$.

(l) If $y = a\cos(\log x) + b\sin(\log x)$, then prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

(m) Find the envelope of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$, m being the parameter.

(n) Show that the function

$$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

has a maximum at (4, 0).

(o) Prove that

$$\int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dx = -\frac{1}{2}$$

(6 MARKS QUESTIONS)

2016

4. *(a)* If

$$f(u, v) = uv\frac{u^2 - v^2}{u^2 + v^2}, \text{ when } u^2 + v^2 \neq 0$$

$$= 0 \quad , \text{ when } \quad u = v = 0$$

then show that $f_{uv}(0, 0) \neq f_{vu}(0, 0)$.

Or

Show that the function

$$f(x, y) = 4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$$

is minimum at (0, 0) while the points

$$\left(\frac{3}{2}, -\frac{3}{2}\right) \text{ and } \left(-\frac{3}{2}, \frac{3}{2}\right)$$

are saddle points of f.

(b) If ρ_1 and ρ_2 be the radii of curvature at the ends M and N of the conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that

$$\rho_1^{2/3} + \rho_2^{2/3} = \frac{a^2 + b^2}{(ab)^{2/3}}$$ 6

Or

Find the asymptotes of the curve

$$y^3 - 6xy^2 + 11x^2y - 6x^3 + y^2 - x^2 + 2x - 3y - 1 = 0$$ 6

5. (a) Obtain a reduction formula for

$$I_{m,\,n} = \int \cos^m x \sin^n x\,dx$$

where m and n are positive integers. Hence, find a reduction formula for

$$\int_0^{\pi/2} \cos^m x \sin^n x\,dx$$ 6

Or

Obtain the area common to the cardioid $r = 2a(1+\cos\theta)$ and the circle $r = 3a$, and also the area of the remainder of the cardioid. 6

(b) The part of the parabola $y^2 = 2ax$ bounded by the latus rectum revolves about the tangent at the vertex. Find the volume and the area of the curved surface of the reel thus generated. 6

Or

Find the volume and surface area of the solid generated by revolving the cardioid $r = 2a(1+\cos\theta)$ about the initial line. 6

6. (a) Prove that $e^{\int P dx}$ is an integrating factor of the linear equation

$$\frac{dy}{dx} + P_y = Q$$

P and Q are the functions of x alone or constants. Hence or otherwise solve the equation

$$e^{x^3}\frac{dy}{dx} + 3x^2 e^{x^3} y = 4x^3$$ 6

Or

If $f(z)\,dx - xz\log z\,dy - xy\,dz = 0$ is integrable, determine $f(z)$ and also, find the solution of the differential equation. 6

(b) Solve : 6

$$(D^2 - 4D + 1)y = e^{2x}(x^2 + \sin 2x)$$

Or

Solve : 6

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 3x\log x$$

2017

4. (a) State and prove Rolle's theorem. 6

Or

State and prove Taylor's theorem with Lagrange's form of remainder. 6

(b) Show that for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the radius of curvature at an extremity of the major axis is equal to half the latus rectum. 6

Or

If ρ_1 and ρ_2 be the radii of curvature at the ends of a focal chord of the parabola $y^2=4ax$, then prove that

$$\rho_1^{-2/3}+\rho_2^{-2/3}=(2a)^{-2/3}$$ 6

5. (a) If $f(x)$ and $g(x)$ are integrable in $[a, b]$ and if $g(x)$ maintains the same sign throughout $[a, b]$, then prove that

$$\int_a^b f(x)\,g(x)\,dx=\mu\int_a^b g(x)\,dx$$

where $m\le\mu\le M$, m and M being the lower and upper bounds of $f(x)$ in $[a, b]$. 6

Or

State and prove the fundamental theorem of integral calculus. 6

(b) Evaluate $\iint\sqrt{4a^2-x^2-y^2}\,dx\,dy$ taken over the upper half of the circle $x^2+y^2-2ax=0$. 6

Or

If u, v, w are the roots of the equation in λ, such that $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)}=\frac{-2(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$ 6

6. (a) If

$$\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}=f(x)$$

then prove that $e^{\int f(x)\,dx}$ is an integrating factor of the equation $Pdx+Qdy=0$. 6

Or

Obtain the condition for integrability of the total differential equation

$$Pdx+Qdy+Rdz=0$$ 6

(b) Find the power series solution of

$$(1-x^2)\frac{d^2y}{dx^2}+2y=0$$ 6

Or

Solve : $(x^2D^2+xD-1)y=\sin(\log x)$ 6

4. a) Show that $\frac{b-a}{1+b^2} \langle \tan$ **2018** $\frac{b-a}{1+a^2}$, $0 \langle a \langle b$ and deduce that $\frac{\pi}{4}+\frac{3}{25} \langle \tan^{-1}\frac{4}{3} \langle \frac{\pi}{4}+\frac{1}{6}$. 6

OR

Show that $\lim_{h \to 0} \theta = \frac{1}{n+1}$, where θ is given by $f(a+h) = f(a)+hf'(a) + \frac{h^2}{\lfloor 2} f''(a)+.....+\frac{h^{n-1}}{\lfloor n-1} f^{n-1}(a)+\frac{h^n}{\lfloor n} f^n(a+\theta h)$ provided $f^{n+1}(x)$ is continuous at a and $f^{n+1}(a) \neq 0$.

b) Show that for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the radius of curvatureat any point ρ is given by $\rho = \frac{a^2b^2}{p^3}$, where p is the perpendicular from the centre on the tangent at ρ. 6

OR

Find the radius of curvature of the curve $y = xe^{-x}$ at its maximum point.

5. a) Evaluate $\int_a^b \cos x dx$ by the method of summation. 6

OR

Evaluate: $\lim_{n \to \infty} \left\{\left(1+\frac{1^2}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)...\left(1+\frac{n^2}{n^2}\right)\right\}^{1/n}$.

5. b) By changing the order of integration, prove that

$$\int_0^1 dx \int_x^{1/x} \frac{y^2 dy}{(x+y)^2 \sqrt{1+y^2}} = \sqrt{2}-\frac{1}{2}.$$ 6

OR

If α, β, γ are the roots of the equation in t, such that

$$\frac{u}{a+t}+\frac{v}{b+t}+\frac{w}{c+t}=1,$$

prove that $\frac{\partial(u,v,w)}{\partial(\alpha,\beta,\gamma)} = -\frac{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}{(a-b)(b-c)(c-a)}$.

6. a) Show that the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal. 6

OR

Obtain the condition for exactness of the differential equation $Mdx + Ndy = 0$.

b) Solve: $(x^2D^2 + xD + 1)y = \sin(\log x^2)$. 6

OR

Find power series solution of the initial value problem $(1-x^2)y'' + 2y = 0$, $y(2) = 4$, $y'(2) = 5$.

4. (a) State and prove Cauchy's mean value theorem. 6

Or

State and prove Rolle's theorem.

(b) Show that radius of curvature at any point of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to three times the length of the perpendicular from the origin to the tangent. 6

Or

Show that for the curve $y = \frac{ax}{a+x}$

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$$

5. (a) State and prove fundamental theorem of integral calculus. 6

Or

If M and m are the least upper and greatest lower bounds of the integrable function $f(x)$ in $[a, b]$, $b > a$, then prove that $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$.

(b) By changing the order of integration, prove that

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}} = \frac{\pi}{2}\log\left(\frac{2e}{1+e}\right)$$ 6

Or

If $u = xyz$, $v = x^2+y^2+z^2$, $w = x+y+z$, then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

6. (a) Show that the family of parabolas $y^2 = 4a(x+a)$ is self-orthogonal. 6

Or

Find $f(y)$, if $f(y)\,dx - zx\,dy - xy\log y\,dz = 0$ is integrable. Find the corresponding integral.

(b) Solve : 6

$$(x+2)^2\frac{d^2y}{dx^2} - 4(x+2)\frac{dy}{dx} + 6y = x$$

Or

Find the power series solution of

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = x$$