DEPARTMENT OF PHYSICS
NAMBOL L. SANOI COLLEGE, NAMBOL

Question Bank (2017- 2019)
Physics
(Honours)
Eighth Paper
PHY: SE-608 (Quantum Mechanics)

Unit I: Origin of Quantum Theory

5 mark Questions

1. What do you understand by the first Bohr radius of hydrogen atom? Find the speed of electron in the first Bohr orbit of hydrogen atom and hence define fine structure constant. 2+2+1=5 (2017)
2. What do you mean by a matter wave? Name an experiment that supports the wave nature of particles. A cricket ball of mass 60 g moves with a speed of 50 m per second. Explain why one cannot observe a de Broglie wave of the cricket ball. (1+1+3=5) (2017)
3. Prove that for rotational motion of a microscopic particle, the uncertainty relation can be stated in the form $\Delta L\ \Delta\Theta \approx \hbar$, where ΔL is the uncertainty in the angular momentum of the particle and $\Delta\Theta$ is the uncertainty in its angular position. (2017)
4. Show that for large n, in a transition of electron from level n to level n - 1 in Bohr model of hydrogen atom, the frequency of radiation approaches the classical frequency of revolution of electron in the nth circular orbit. (2017)
5. Define phase velocity and group velocity of a wave packet. Prove that the group velocity of a wave packet is equal to the particle velocity of the associated particle.(1+1+3=5)(2017)
6. What is Compton Effect? Explain why Compton shift is not observed with visible light. (3+2=5) (2019)
7. How do you understand by Planck constant? In the following, match List I (Fundamental experiments) with List II (Conclusions) appropriate pairs :

List-I	List-II
(i) Photoelectric effect	1. Quantization of atomic energy levels
(ii) Franck-Hertz experiment	2. Wave nature of electron
(iii) Davisson-Germer experiment	3. Particle nature of electromagnetic radiation

(2019)

8. State Bohr's postulate of angular momentum quantization in his atom model. Prove that de Broglie's hypothesis of matter waves supports Bohr's angular momentum quantization in his atom model. (2+3=5) (2019)
9. What is the basic difference between electromagnetic waves and matter waves? Although electron diffraction and neutron diffraction are both used in the study of structure of materials, argue that neutron diffraction is superior to X-ray diffraction in the study of protein structure. (2+3=5) (2019)
10. What do you mean by ultraviolet catastrophe? Show how Rayleigh-Jeans formula of blackbody radiation leads to ultraviolet catastrophe. ($2\frac{1}{2}$+$2\frac{1}{2}$=5) (2020)
11. In an experiment on photoelectric effect, if V_I and V_{II} are the stopping potentials corresponding to lights of wavelengths A and Ag respectively, prove that

    i. $h = \frac{e(V_{II}-V_I)\lambda_I\lambda_{II}}{c(\lambda_I-\lambda_{II})}$

    ii. $h = \frac{e(V_{II}\lambda_{II}-V_I\lambda_I)}{\lambda_I-\lambda_{II}}$

    where the symbols have their usual meanings. ($2\frac{1}{2}$+$2\frac{1}{2}$=5) (2020)

12. In Bohr model of hydrogen atom, obtain the expression for the orbiting speed of electron in the first orbit and hence define the fine-structure constant α (alpha).(4+1=5) (2020)
13. State Wilson-Sommerfeld quantization rule and apply the rule to find the energy spectrum for a particle of mass m confined in a one-dimensional box ($0 \leq x \leq a$).(2+3=5) (2020)
14. Write down Heisenberg's position momentum uncertainty relation and use it to estimate the Bohr radius of hydrogen atom.(1+4=5) (2020)

10 mark Questions

1. In Compton Effect, an incident photon of frequency ν_0 is scattered at an angle θ by a stationary electron. If the electron is recoiled at an angle Φ with the direction of incident photon, establish the relation

$$\tan\phi = \frac{1}{1+\frac{h\nu_0}{mc^2}}\cot\frac{\theta}{2}$$

where ***m*** is rest mass of electron, ***c*** is speed of light in free space and ***h*** is Planck constant. Derive an expression for kinetic energy of the recoiled electron. (6+4=10) (2017)

2. Consider Compton Effect in which a photon of frequency ν_0 and wavelength λ_0 is scattered by a free electron moving with a linear momentum of magnitude p in the same direction as that of the incident photon. Show that the Compton shift in wavelength of photon is given by

$$\Delta\lambda = \frac{2\lambda_0(p_0+p)c}{E-pc}\sin^2\left(\frac{\theta}{2}\right)$$

where $p_0 = \frac{h}{\lambda_0}$ is the magnitude of incident photon momentum, **ϴ** is the photon scattering angle, **E** is the initial relativistic energy of the electron and **c** is the speed of light in free space. (2017) (2020)

3. Give reason why Bohr atom model is known as a semi-classical model. Using Bohr-Wilson-Sommerfeld quantization rule, obtain the energy spectrum of a linear harmonic oscillator. (2+8=10) (2019)
4. State Bohr's correspondence principle and verify the principle in the case of lBohr's model of a hydrogenic atom. (2+8=10) (2019)
5. State the basic differences between photoelectric effect and Compton Effect. A photon of frequency ν_o is Compton-scattered by an electron at rest and the scattered photon of frequency ν moves in a direction perpendicular to the direction of incident photon. Prove that the de Broglie wavelength of the recoil electron is given by

$$\lambda_r = \frac{c}{\sqrt{\nu_o^2 + \nu^2}}$$

where c is the speed of light in free space. (3+7=10) (2020)

Unit II: Basic postulates and formalism

3 mark Questions

1. The wave function of a particle, constrained to move in the x-direction from -∞ to +∞, is defined by

$$\boldsymbol{\Psi(x) = A e^{-\frac{x^2}{a^2} + ibx}}$$

where A, a and b are real numbers. If the wave function is normalized, find the value of A. (2017)

2. Show that the function

$$\boldsymbol{\Psi(x) = x e^{-\frac{x^2}{2}}}$$

is an eigen function of the operator ($\mathbf{x^2 - d^2/dx^2}$) with an eigen value 3. (2017)

3. Considering one-dimensional case, prove that the momentum operator is Hermitian. (2017)
4. For a function f(x), prove that

$$[\boldsymbol{f(x), p}] = i\hbar \frac{d\boldsymbol{f}}{d\boldsymbol{x}}$$

where p is the momentum operator in one dimension. (2017)

5 mark Questions

1. What do you understand by superposition theorem in quantum mechanics? The ground state and the first excited state of a quantum mechanical system are described by the wave functions $\boldsymbol{\Psi}_o$(x, t), and $\boldsymbol{\Psi}_1$ (x, t) respectively. Using the superposition theorem, construct another possible wave function. What will be the new probability density? (2+1+2 =5) (2019)

6 mark Questions

1. If a state of a quantum mechanical system is described by a wave function w(x, t), define probability density and probability current density. Hence derive probability current density continuity equation.
2. What is the physical significance of normalization of a wave function? The wave function of a particle constrained to move along the x-axis from -∞ to +∞ at a certain instant is given

$$\boldsymbol{\Psi(x) = A e^{-\frac{x^2}{a^2} + ibx}}$$

where A, a and b are real numbers. If the wave function is normalized, find the value of A. (2+4=6) (2019)

3. State the postulates of quantum mechanics citing the physical interpretation of expansion coefficients.
4. Prove that (i) eigenvalues of a Hermitian operator are real and (ii) eigenfunctions of a Hermitian operator belonging to different eigenvalues are orthogonal. 3+3=6 (2019)
5. What do you mean by the commutator of two operators A and B? Prove the Jacobi identity of commutators in the case of three operators A, B and C. (2+4=6) (2019)
6. Prove that $\boldsymbol{[x^n, p] = i\hbar n x^{n-1}}$, where x and p are one-dimensional position and momentum operators. Hence, deduce that $[\boldsymbol{f(x)}, \boldsymbol{p}] = i\hbar \frac{df}{dx}$, where f(x) is a polynomial in x. (5+1=6) (2019)
7. A non-relativistic particle of mass m is moving in the x-direction in a force field derivable from a potential V(x) which does not depend explicitly on time t. If the particle is described by a wave function $\Psi(x,t)$= $Ae^{i(kx-wt)}$, where A is an arbitrary real constant, k is the propagation constant and ω the angular frequency, develop the one-dimensional time-dependent Schrödinger equation in the form

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

8. The wave function of a particle is given by $\Psi(x,t)$= $Ae^{-\alpha(x-\beta)2}$, where A is a real constant known as normalization constant and a and ß are also real constants. Find the (i) constant A and (ii) point where the particle is most likely to be found. (3+3=6) (2020)
9. Find the position probability density and the probability current density if the wave function of a particle is given by

$$\Psi(x,t) = A\sin kx e^{-\frac{i\hbar k^2 t}{2m}}$$

where A is a constant. (2+4=6) (2020)

10. State Ehrenfest's theorem. Prove that

$$\frac{d\langle x\rangle}{dt} = \frac{1}{m} < p_x >$$

where the symbols have their usual meanings. (1+5=6) (2020)

11. Derive the quantum mechanical equation of motion of the expectation value of a dynamical variable and hence show that for a conservative system, the total energy is a constant of motion. 4+2-6 (2020)
12. Define a Hermitian operator. If the one-dimensional momentum operator is defined as p= Px=- $i\hbar\frac{df}{dx}$, prove that

    $[x^n, p] = i\hbar n x^{n-1}$ (2+4-6) (2020)

7 mark Questions

1. If a physical system with a potential V(r) is described by a wave function $\boldsymbol{\Psi}$ (r, t), derive an expression for probability current density. Deduce that probability current density vanishes if the wave function is real. (5+2=7) (2017)
2. Considering a wave function

$$\boldsymbol{\Psi}(\boldsymbol{x},\boldsymbol{t}) = \boldsymbol{A}e^{\boldsymbol{i}(\boldsymbol{kx}-\boldsymbol{wt})}$$

   Where A is a constant and k and ω are wave number and angular frequency of the associated wave, develop time-dependent Schrödinger equation. What do you mean by a normalized wave function? 6+1=7 (2017)
3. What do you mean by a Hermitian operator? Prove that
   i. eigenvalues of a Hermitian operator are real;
   ii. eigenfunctions belonging to different eigenvalues of a Hermitian operator are orthogonal. (1+3+3=7) (2017)
4. What is a linear operator? State the fundamental postulates of quantum mechanics.1+6=7 (2017)

Unit III: Stationary states and Energy eigen states, Particle in one dimensional box, Linear Harmonic Oscillator, One dimensional potential barrier, Hydrogen atom

5 mark Questions

1. The lowest entrapped energy of a particle in a one-dimensional force-free region of length a is found to be 40 eV. Write down a general expression for energy of all possible states. Find the energies in the next four higher states.(1+4=5) (2017)

2. The eigen function of a linear harmonic oscillator in the ground state is defined by

$$u_0(x) = \left(\frac{a}{\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{a^2x^2}{2}} \text{ where } a = \sqrt{\frac{m\omega}{\hbar}}$$

Prove that the probability of finding the oscillator beyond the classical limit is approximately 17%. (2017)

3. An electron beam of energy 36 eV is incident on a potential step of height 16 eV. Define a reflection coefficient and calculate its magnitude. (2+3=5) (2017)

8 mark Questions

1. Obtain a plane wave solution of time-dependent Schrödinger equation for a free particle moving in the x-direction with a velocity v. Prove that the probability current density in this particular problem is equal to the particle velocity multiplied by the position probability density. 5+3=8 (2017)
2. A particle of mass m and energy E moving from left to right in a potential-free region is incident on a potential step at x=0. The potential step is defined by

$$V(x) = \begin{cases} 0 & \text{for} \quad x<0 \\ V_o & \text{for} \quad x>0 \end{cases}$$

Solve time-independent Schrödinger equations in the two regions with the relevant boundary conditions. Prove that for $E<V_o$, the position probability density exhibits oscillatory behaviour in the region x<0 and also prove that there is a finite probability that the particle can enter the classically inaccessible region (x > 0). 4+2+2=8 (2017)
3. In a hydrogen atom, the electron is in the 1s state. Write down the corresponding radial wave function and find the expectation value of $\frac{1}{r}$ (r being T the distance of electron from the proton). What will be the expectation value of potential energy? In the given state, is there any degeneracy? 1+5+1+1=8 (2019)
4. Write down the complete ground state wave function Ψ_{100} (r, ϴ, Φ) of a hydrogen atom. Prove that the most probable distance of electron from the nucleus in the ground state of hydrogen atom is equal to the Bohr radius. Can the principal quantum number n for an electron in a hydrogen atom have zero value? Justify your answer. 1+5+1+1=8 (2019)
5. What do you understand by reduced mass of two particles? Using the reduced mass of nucleus and electron of a hydrogenic atom, write down the time-independent Schrödinger equation and separate it into radial and angular parts. 2+6=8 (2019)
6. Define a stationary state and a free particle. If a plane wave is described by a wave-function $\Psi(x,t) = Ae^{i(kx-wt)}$, where A is a constant and k and w are propagation constant and an k and w are propagation constant and angular frequency respectively, prove that the probability current density is given by position probability density times the velocity of propagation. (3+5 =8) (2020)

7. A particle is confined to move in a one dimensional infinite square well defined by the potential function

$$V(x) = = \begin{cases} 0 & \text{for} \quad -a<x<a \\ \infty & \text{otherwise} \end{cases}$$

Solve the relevant time-independent Schrödinger equation using the necessary boundary conditions to show that the normalized eigenfunctions are

$$\Psi_n(x) = \begin{cases} \frac{1}{\sqrt{a}}\cos\left(\frac{n\pi x}{2a}\right) & \text{for } n = 1, 3, 5, \ldots \\ \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi x}{2a}\right) & \text{for } n = 2, 4, 6, \ldots \end{cases}$$

8. A particle having energy E and moving from left to right is incident on a potential step V_o (with E> V_o) at x = 0. Solving the relevant time-independent Schrödinger equation, prove that there exists a transmission coefficient given by $T = \frac{4n}{(1+n)^2}$, where n is the refractive index of the medium in region with V = V_o relative to that with V=0. Sketch a schematic plot of T as function of $\frac{E}{V_O}$. (6+2=8) (2020)
9.

10 mark Questions

1. The complete wave function of a hydrogenic atom corresponding to an energy eigen value E, is designated as $\boldsymbol{\Psi}_{nlm}$ (r, ϴ, Φ) where the symbols have usual meanings. Assigning possible values to I and m, show that the state corresponding to energy eigen value E_2 is four-fold degenerate. Write down the ground-state radial wave function R and calculate the expectation value of 1/r where r electron. is radial distance of electron. (4+1+5=10) (2017)

2. Write down the complete ground-state wave function W100 of a hydrogen atom and find out the position probability density. Also, show that the most probable distance of electron from the nucleus in the ground state of hydrogen atom is equal to the Bohr radius. Can the principal quantum number n for an electron in a hydrogen atom have zero value? Justify your answer. (1+3+3+1+2=10) (2017)

12 mark Questions

1. A particle moving in one dimension is confined to move within a region bounded by x=0 and x = L under a potential given by

$$V(x) = \begin{cases} 0 & \text{for } 0 \le x \le L \\ \infty & \text{for } x<0 \text{ and } x > L \end{cases}$$

   i. Obtain normalized eigen functions of the problem. **5**

ii. Show that

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{L^2}{12}\left(1 - \frac{6}{\pi^2 n^2}\right) \qquad 5$$

iii. Is the wave function continuous everywhere in the region? 1

iv. Is the derivative of the wave function continuous everywhere in the region? 1 (2017)

2. State virial theorem. If x and p represent the position and momentum operators, using an eigen function of the nth state of a linear harmonic oscillator, obtain the expectation value of x^2 and p^2 and hence verify the virial theorem. (2+4+4+2=12) (2017)

3. State virial theorem. If x and p represent the position and momentum operators, using the ground state eigenfunction of a linear harmonic oscillator, obtain the expectation values of x^2 and p^2, and hence verify the virial theorem. (2+3+5+2=12) (2019)

4. Write down the time-independent Schrödinger equation of a linear harmonic oscillator and solve it to obtain the eigenfunctions and the energy eigenvalues. (2+10=12) (2019)

5. A particle moving in a one-dimensional region from x = 0 to x = a is in a state represented by a wave function $\Psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\Pi x}{a}\right)$

Find the expectation values $\langle x \rangle, \langle p \rangle, \langle p^2 \rangle$ and $\langle x^2 \rangle$. (3 x 4 =12) (2019)

6.

7. An electron having energy E and moving from left to right is incident on a potential step V_O ($V_O > E$) at x = 0 . Solving the relevant time-independent Schrödinger equation, show that the wave function of the electron within the region of step potential is given by

$$\Psi_{II}(x) = C e^{-\alpha_2 x)}$$

where the subscript II denotes region with potential V_O , C is a constant and

$\alpha_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$, where the symbols have usual meanings. What is quantum tunnelling? (10+2=12)(2019)

8. The normalized nth state eigen function of a linear harmonic oscillator is represented by

$$\Psi_n(x) = \left(\frac{\alpha}{2^n n! \sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\alpha^2 x^2/2} H_n(\alpha x)$$

Prove that the expectation value of position operator is given by (x) = 0 whereas the matrix elements for position operator matrix is given by

$$X_{nm} = \begin{cases} 0, & m \neq n \pm 1 \\ \frac{1}{\alpha}\sqrt{\frac{n+1}{2}}, & m = n+1 \\ \frac{1}{\alpha}\sqrt{\frac{n}{2}}, & m = n-1 \end{cases}$$

(3+9=12) (2020)

9. The eigen functions of a linear harmonic oscillator in the ground state and in the first excited state are respectively represented by

$$\Psi_0(x) = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\alpha^2 x^2/2}$$

$$\Psi_1(x) = \left(\frac{\alpha}{2\sqrt{\pi}}\right)^{\frac{1}{2}} 2\alpha x e^{-\alpha^2 x^2/2}$$

Where $\alpha = \sqrt{\frac{m\omega}{\hbar}}$

i. For the ground state, determine the classical turning points and show that the probability of finding the particle outside the classical turning points is about 16%. (2+4-6)

ii. For the first excited state, find the point where the probability density is maximum and also find the value of the maximum probability density. (3+3=6) (2020)

10. Using the radial eigenfunction of the nth state hydrogen atom, determine the expectation value of $\frac{1}{r}$ and hence of the potential V(r) and the kinetic energy $\frac{p^2}{2m}$, stating a generalized Virial theorem, verify the theorem. (6+2+4=12) (2020)
11. Define the terms (i) hydrogen-like atom and (ü) Bohr radius. Using the radial eigen function of the nth state hydrogen atom, determine the expectation value of $\frac{1}{r^2}$ (2+10=12) (2020)